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TIME FREQUENCY ANALYSIS OF A NOISY CARRIER SIGNAL

by

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Digital communication signals are inherently nonstationary (time varying spectral content) signals due to their information content. This suggests the use of time frequency analysis methods for detecting these signals as well as obtaining the desired signal parameters. For signals employing *phase-shift-keying* (PSK) to convey information, the important signal parameters are the carrier (sinusoid) center frequency and information (bit) rate.

In this thesis, the signal detection problem is addressed using time frequency processing of a carrier signal embedded in additive white gaussian noise (AWGN). A TFR (time frequency representation) performance measure based on a mean and variance analysis is proposed and used to estimate the center frequency of a carrier. Through computation of the discrete-time TFR, our results show that this measure provides a means to determine the presence of a carrier signal in noise even when the TFR itself becomes quite obscured by the noise. The cone kernel-TFR is seen to yield the highest frequency resolving capability compared with the Wigner-Ville distribution and the Choi-Williams distribution.

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TIME FREQUENCY ANALYSIS OF A NOISY CARRIER SIGNAL

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Digital communication signals are inherently nonstationary (time varying spectral content) signals due to their information content. This suggests the use of time frequency analysis methods for detecting these signals as well as obtaining the desired signal parameters. For signals employing *phase-shift-keying* (PSK) to convey information, the important signal parameters are the carrier (sinusoid) center frequency and information (bit) rate.

In this thesis, the signal detection problem is addressed using time frequency processing of a carrier signal embedded in additive white gaussian noise (AWGN). A TFR (time frequency representation) performance measure based on a mean and variance analysis is proposed and used to estimate the center frequency of a carrier. Through computation of the discrete-time TFR, our results show that this measure provides a means to determine the presence of a carrier signal in noise even when the TFR itself becomes quite obscured by the noise. The cone kernel-TFR is seen to yield the highest frequency resolving capability compared with the Wigner-Ville distribution and the Choi-Williams distribution.

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I. INTRODUCTION

A. MOTIVATION

In digital communication systems employing *binary phase-shift-keying* (BPSK) to convey information (bits), the phase of a carrier (sinusoid) signal is shifted by 0 degrees or 180 degrees using a digital waveform. The digital waveform is a bipolar voltage signal generated by the bit stream representing the data, where a bit "one" might correspond to "positive" and a bit "zero" to "negative" (or vice versa). Therefore, the BPSK signal is simply a carrier signal multiplied (modulated) by a data waveform. This results in a nonstationary (time varying spectral content) signal, which suggests the use of time frequency analysis methods for determining both the signal's spectral *and* temporal information.

Suppose one is interested in the detection of a BPSK signal in noise. An intended receiver of a BPSK signal simply uses a correlation receiver which consists of a local oscillator (i.e., carrier center frequency), an integrator (over one bit period), and a comparator. On the other hand, an *un*intended receiver of a BPSK signal must use some other means of detection since he lacks knowledge of the vital signal parameters. In the absence of noise, a TFR (time frequency representation) of a BPSK signal would reveal information about the carrier center frequency, the bit rate, *and* the times at which the phase shifts occurred. However, the inevitable presence of noise in a communications channel calls for an examination into the ability of a TFR to detect nonstationary signals in noise.

A direct sequence spread spectrum (DS-SS) signal represents a more realistic signal targeted for interception by a *un*intended receiver. Since a DS-SS signal is itself a PSK signal, the foregoing discussion related to using the TFR for detecting such signals applies, except that a parameter analogous to the bit rate termed the *chip* rate is the parameter of interest (in addition to the carrier center frequency). Through the "spectrum spreading" process, the spectrum of the underlying BPSK signal is reduced in amplitude below the noise level. It is for this reason that an analysis of the performance of a TFR used for detecting a signal *in noise* is to be examined.

For this thesis, our analysis is restricted to the performance evaluation of a TFR used to detect the center frequency of a noisy carrier signal.

B. TIME FREQUENCY ANALYSIS

Time frequency analysis of signals result in a representation of a signal's energy jointly in time and frequency. The resulting time frequency representation (TFR) is therefore a depiction of the spectral content of a signal as a function of time. An infinite number of TFR's can be generated for a particular signal using a "generalized" expression for the TFR (GTFR) known as Cohen's class of bilinear TFR's [Ref. 1]. Specifically, a TFR within this class is completely specified by a two-dimensional function called the "kernel" function according to the following expression:

$$TF(t,f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(\nu,\tau)x(u+\tau/2)x^*(u-\tau/2)e^{-j2\pi(\nu t-\nu u+f\tau)}du\,d\tau\,d\nu$$

where $x(t)$ is the signal of interest and $\phi(\nu,\tau)$ is the kernel function.[1]

[1] $[\;]^*$ denotes complex conjugate.

The TFR can best understood as a two-dimensional Fourier transform of the product of the kernel function with the auto-ambiguity function, $A(\nu,\tau)$, defined as

$$A(\nu,\tau) = \int_{-\infty}^{\infty} x(u+\tau/2)x^*(u-\tau/2)e^{-j2\pi\nu u}du$$

However, since the order of integration can be interchanged, the TFR can be also be interpreted as a two-dimensional filtered Wigner-Ville distribution [Ref. 2: p38] where the filter is the kernel defined in the time-frequency plane. [Ref. 2] provides a detailed summary of the properties of the Wigner-Ville distribution since it does represent the most fundamental TFR within Cohen's class. These are just two of many interpretations of the TFR, some of which can provide insight into the selection of a kernel function for a particular signal.

Since there is no "best" TFR for any given signal, there has been an effort to provide a method for determining an "optimal" kernel for an arbitrary signal [Ref. 3]. The basic approach used in [Ref. 3] was to eliminate as much of the cross terms in the auto-ambiguity function of a signal while retaining as much of the auto-components as possible. [Refs. 4-6] proposed and examined the use of a cone-shaped kernel for the processing of PSK signals in addition to speech signals. In [Ref. 5], the use of the cone kernel TFR (CK-TFR) for PSK signals resulted in a spectral "peak-splitting" effect coincident with the phase shifts occurring in the signal. Since the cone kernel yielded superior time and frequency resolution for a PSK signal compared to the spectrogram [Ref. 5], it may be considered a better choice of kernel for detecting and analyzing PSK signals.

C. OUTLINE

In this thesis, the signal detection problem is addressed using time frequency processing. We would like to determine whether there is a better kernel (i.e., TFR) to use in time frequency analysis for the detection of a carrier signal embedded in AWGN. Toward this end, a decision statistic and a related TFR performance measure is proposed in Chapter II for a TFR. Using the discrete-time formulation of the TFR, the equations used to calculate the mean and variance of the decision statistic are derived. Chapter III is devoted to the Wigner-Ville Distribution (WVD) of a carrier signal. In Chapter IV, the equations developed in Chapters II and III are implemented for three different kernels. In particular, the proposed performance measure using the WVD is computed for various noise levels. Additionally, a comparison of the results obtained using three different kernels is made. Finally, a summary of the results and final remarks are given in Chapter V.

II. TIME FREQUENCY ANALYSIS OF A DISCRETE-TIME SIGNAL

Signal analysis using a time-frequency representation (TFR) cannot be easily depicted in a closed form expression. In order to facilitate practical signal analyses, the discrete-time TFR's are employed for analyzing discrete-time signals. In this chapter, a TFR performance measure based on a mean and variance analysis is proposed and used to estimate the frequency of a carrier signal (sinusoid) embedded in AWGN.

A. SIGNAL SEQUENCE

For practical signal analysis, only a finite amount of data can be used or is available for use by the signal analyst, usually in the form of a discrete sequence. Since the time origin can be arbitrarily chosen, an arbitrary continuous-time signal, $x(t)$, with a duration of T seconds can be assumed to exist in the interval $[-T/2, T/2]$ for analytical convenience. If $x(t)$ is sampled at regular intervals of duration T_s seconds, the resulting sequence,

$$x(n) = [x(-L), x(-L+1), \ldots, x(0), \ldots, x(L)]$$

is defined for n in the interval $[-L, L]$, where $T = 2LT_s$ and the total number of samples is $M = 2L+1$.

B. DISCRETE-TIME TFR

The TFR of the continuous-time signal $x(t)$ is given as

$$TF(t,f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(t-t',\tau)x(t'+\tau/2)x^{*}(t'-\tau/2)e^{-j2\pi f\tau}\,dt'\,d\tau$$

where $\phi(n,k)$ is the kernel function and $x(t)$ is the signal. Discretizing the frequency, we obtain the discrete-time TFR as

$$TF(n,m) = T_s \sum_{p=-L}^{L}\sum_{k=-L}^{L} \phi(p,k)x(n-p+k)x^{*}(n-p-k)e^{-j\left(\frac{2\pi}{M}\right)mk} \tag{2.1}$$

where $f = m/(MT_s)$, $M = 2L+1$, and $m = [-L,-L+1,\ldots,0,\ldots,L]$. Note that transforming the continuous-time variable $\tau/2$ to the discrete-time variable k implies that the frequencies in the resulting TFR are multiplied by a factor of two.

If $x(n)$ is a real sequence and $\phi(n,k)$ is a real, even function in k (i.e. $\phi(n,k) = \phi(n,-k)$), then

$$\begin{aligned}
TF^{*}(n,m) &= T_s \sum_{k=-L}^{L}\sum_{p=-L}^{L} \phi(p,k)x(n-p+k)x(n-p-k)e^{+j\left(\frac{2\pi}{M}\right)mk} \\
&= T_s \sum_{k=-L}^{L}\sum_{p=-L}^{L} \phi(p,-k)x(n-p-k)x(n-p+k)e^{+j\left(\frac{2\pi}{M}\right)mk} \\
&= T_s \sum_{l=-L}^{L}\sum_{p=-L}^{L} \phi(p,l)x(n-p+l)x(n-p-l)e^{-j\left(\frac{2\pi}{M}\right)ml} \\
&= TF(n,m),
\end{aligned}$$

which is real, and therefore

$$TF(n,m) = \frac{1}{2}[TF(n,m) + TF^*(n,m)]$$

$$= \frac{T_s}{2}\sum_{k=-L}^{L}\sum_{p=-L}^{L}\phi(p,k)x(n-p+k)x(n-p-k)\left(e^{-j\left(\frac{2\pi}{M}\right)mk} + e^{+j\left(\frac{2\pi}{M}\right)mk}\right)$$

$$= T_s\sum_{k=-L}^{L}\sum_{p=-L}^{L}\phi(p,k)x(n-p+k)x(n-p-k)\cos(2\pi mk / M) \tag{2.2}$$

Finally, since (2.2) is an even function in m ($TF(n,m) = TF(n,-m)$), only values of $TF(n,m)$ for m in the interval $[0, L]$ need be used for displaying the TFR.

C. DETECTING THE CENTER FREQUENCY OF A CARRIER SIGNAL

Suppose $x(n) = A\cos(2\pi m_0 n / M)$ for $|n| \le L$, and we are interested in determining the carrier frequency (specified by m_0) from its TFR. Since the TFR can be considered as a joint distribution of a signal's energy over the time-frequency plane, summing $TF(n,m)$ over all n effectively collects the total energy contained within each frequency bin. We define a measure, or a decision statistic, $Y(m)$ as:

$$Y(m) = \sum_{n=-\infty}^{\infty} TF(n,m) \tag{2.3}$$

One would expect $Y(m)$ to have a maximum value located at $m = m_0$. If equation (2.2) is substituted into (2.3), we have

$$Y(m) = T_s\sum_{k=-L}^{L}\sum_{p=-L}^{L}\phi(p,k)\left[\sum_{n=-\infty}^{\infty}x(n-p+k)x(n-p-k)\right]\cos(2\pi mk / M)$$

and if we make the substitution, $l = n - p$, and observe that $TF(n,m)$ is defined only for n in the interval $[-2L, 2L]$ since $x(n)$ is defined only for $|n| \le L$, we obtain

$$= T_s \sum_{k=-L}^{L} \sum_{p=-L}^{L} \phi(p,k) \left[\sum_{l=-2L-p}^{2L-p} x(l+k)x(l-k) \right] \cos(2\pi mk / M)$$

Notice that $x(l+k)x(l-k)$ is defined only when $-L \le l+k \le L$, *and* $-L \le l-k \le L$.

Therefore, we have

$$-L+|k| \le l \le L-|k| \quad \Rightarrow \quad -L \le l \le L, \quad \forall\, k,p \in [-L,L]$$

which yields

$$Y(m) = T_s \sum_{k=-L}^{L} \sum_{n=-L}^{L} \left[\sum_{p=-L}^{L} \phi(p,k) \right] x(n+k)x(n-k) \cos(2\pi mk / M)$$

$$= T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) x(n+k)x(n-k) \cos(2\pi mk / M) \qquad m = [0,1,\ldots,L] \tag{2.4}$$

where

$$\rho(k) = \sum_{p=-L}^{L} \phi(p,k) \tag{2.5}$$

D. NOISE ANALYSIS

Consider a deterministic signal sequence, $s(n)$, embedded in an AWGN noise sequence, $w(n)$. We set

$$x(n) = \begin{cases} s(n) + w(n), & |n| \le L \\ 0, & otherwise \end{cases}$$

with $\mathbf{E}\{w(n)\} = 0$, and $\mathbf{E}\{w(n_1)w(n_2)\} = \sigma^2 \delta(n_1 - n_2)$, where

$$\delta(n_1 - n_2) = \begin{cases} 1, & n_1 = n_2 \\ 0, & n_1 \ne n_2 \end{cases}$$

and $\mathbf{E}\{\bullet\}$ denotes expectation. Now, if we substitute $x(n) = s(n) + w(n)$ into (2.4), we get

$$Y(m) = T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k)[s(n+k) + w(n+k)][s(n-k) + w(n-k)]\cos(2\pi mk / M)$$

$$\begin{aligned} &= T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) s(n+k) s(n-k) \cos(2\pi mk / M) \\ &\quad + T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) w(n+k) w(n-k) \cos(2\pi mk / M) \\ &\quad + T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) s(n+k) w(n-k) \cos(2\pi mk / M) \\ &\quad + T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) w(n+k) s(n-k) \cos(2\pi mk / M) \end{aligned} \tag{2.6}$$

For simplicity of analysis, we assume that $\rho(k)$ is even in k. It follows that

$$T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) s(n+k) w(n-k) \cos(2\pi mk / M)$$

$$= T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(-k) s(n-k) w(n+k) \cos(2\pi m(-k) / M)$$

$$= T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) s(n-k) w(n+k) \cos(2\pi mk / M)$$

so the last two terms in (2.6) are the same. Therefore, we have

$$Y(m) = Y_s(m) + Y_w(m) + Y_{cross}(m) \tag{2.7}$$

where

$$Y_s(m) = T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) s(n+k) s(n-k) \cos(2\pi mk / M) \tag{2.8}$$

$$Y_w(m) = T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) w(n+k) w(n-k) \cos(2\pi mk / M) \tag{2.9}$$

$$Y_{cross}(m) = 2T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) s(n+k) w(n-k) \cos(2\pi mk / M) \tag{2.10}$$

Note that since $Y(m)$ is the sum of a deterministic component, $Y_s(m)$, and random (noise) component, $Y_w(m) + Y_{cross}(m)$, it is a sequence of random variables in frequency (indexed by m). For convenience, we define the random component of $Y(m)$ as

$$Y_{noise}(m) = Y_w(m) + Y_{cross}(m)$$

which gives

$$Y(m) = Y_s(m) + Y_{noise}(m) \tag{2.11}$$

1. Mean Analysis

The mean of $Y(m)$ is found by taking the expectation of (2.11) as follows

$$\mathbf{E}\{Y(m)\} = Y_s(m) + \mathbf{E}\{Y_{noise}(m)\} \tag{2.12}$$

where $\mathbf{E}\{Y_{noise}(m)\} = \mathbf{E}\{Y_w(m)\} + \mathbf{E}\{Y_{cross}(m)\}$. Using (2.9) and taking the expectation inside the summation, we get

$$\begin{aligned}\mathbf{E}\{Y_w(m)\} &= T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k)\, \mathbf{E}\{w(n+k) w(n-k)\} \cos(2\pi mk / M) \\ &= T_s\sigma^2 \sum_{n=-L}^{L} \sum_{k=-L}^{L} \rho(k) \delta(2k) \cos(2\pi mk / M)\end{aligned}$$

since $\mathbf{E}\{w(n+k)w(n-k)\} = \sigma^2 \delta(2k)$. Evaluating the summations, we obtain

$$\mathbf{E}\{Y_w(m)\} = MT_s\sigma^2\rho(0) \tag{2.13}$$

The expected value of (2.10) is easily seen to be zero since $\mathbf{E}\{w(n-k)\} = 0$. Therefore, the only contribution to the mean of $Y(m)$ is from the auto-signal and auto-noise components, $Y_s(m)$ and $Y_w(m)$, respectively. Using (2.13) in (2.12) yields the final expression for the mean of $Y(m)$:

$$\mathbf{E}\{Y(m)\} = Y_s(m) + MT_s\sigma^2\rho(0) \tag{2.14}$$

2. Variance Analysis

Since $Y_s(m)$ is deterministic, we have

$$\mathbf{Var}\{Y(m)\} = \mathbf{Var}\{Y_{noise}(m)\} = \mathbf{E}\{Y_{noise}^2(m)\} - [\mathbf{E}\{Y_{noise}(m)\}]^2 \tag{2.15}$$

where $\mathbf{Var}\{\bullet\}$ denotes variance. Evaluating $Y_{noise}^2(m)$, we get

$$\begin{aligned} Y_{noise}^2(m) &= [Y_w(m) + Y_{cross}(m)]^2 \\ &= Y_w^2(m) + Y_{cross}^2(m) + 2Y_w(m)Y_{cross}(m) \end{aligned}$$

Then, taking expectation of above expression, we get

$$\mathbf{E}\{Y_{noise}^2(m)\} = \mathbf{E}\{Y_w^2(m)\} + \mathbf{E}\{Y_{cross}^2(m)\} + \mathbf{E}\{2Y_w(m)Y_{cross}(m)\}$$

In order to evaluate the above expression, it is necessary to use the following relation for zero mean, jointly normal random variables from [Ref. 7].

$$\begin{aligned} \mathbf{E}\{w_1 w_2 w_3 w_4\} &= C_{12}C_{34} + C_{13}C_{24} + C_{14}C_{23} \\ \text{where } C_{ij} &= \mathbf{E}\{w_i w_j\} \end{aligned} \tag{2.16}$$

We have

$$\mathbf{E}\{2Y_w(m)Y_{cross}(m)\} = 0$$

since this term involves the product of three, zero mean, gaussian random variables. We can show this by observing that if we set $w_4 = 1$ (with certainty) in (2.16), we have

$$\mathbf{E}\{w_1 w_2 w_3\} = C_{12}\,\mathbf{E}\{w_3\} + C_{13}\,\mathbf{E}\{w_2\} + C_{23}\,\mathbf{E}\{w_1\} = 0$$

Therefore, all terms in the summation are zero. We are left to evaluate

$$\mathbf{E}\{Y_{noise}^2(m)\} = \mathbf{E}\{Y_w^2(m)\} + \mathbf{E}\{Y_{cross}^2(m)\} \tag{2.17}$$

If we square both sides of (2.9) and take expectation, we have

$$\mathbf{E}\{Y_w^2(m)\} = T_s^2 \sum_{n=-L}^{L} \sum_{n'=-L}^{L} \sum_{k=-L}^{L} \sum_{k'=-L}^{L} [\rho(k)\rho(k')\cos(2\pi mk/M)\cos(2\pi mk'/M)$$

$$\bullet\, \mathbf{E}\{w(n+k)w(n-k)w(n'+k')w(n'-k')\}]$$

The expectation within the summation is evaluated by applying (2.16) as follows:

$$\begin{aligned}
&\mathbf{E}\{w(n+k)w(n-k)w(n'+k')w(n'-k')\} \\
&\quad = \mathbf{E}\{w(n+k)w(n-k)\}\,\mathbf{E}\{w(n'+k')w(n'-k')\} \\
&\qquad + \mathbf{E}\{w(n+k)w(n'+k')\}\,\mathbf{E}\{w(n-k)w(n'-k')\} \\
&\qquad + \mathbf{E}\{w(n+k)w(n'-k')\}\,\mathbf{E}\{w(n-k)w(n'+k')\} \\
&\quad = \sigma^4\delta(k)\delta(k') \\
&\qquad + \sigma^4\delta(n-n'+k-k')\delta(n-n'-k+k') \\
&\qquad + \sigma^4\delta(n-n'+k+k')\delta(n-n'-k-k') \\
&\quad = \sigma^4[\delta(k)\delta(k') + \delta(n-n')\delta(k-k') + \delta(n-n')\delta(k+k')]
\end{aligned}$$

We substitute this into the expression for $\mathbf{E}\{Y_w^2(m)\}$ and evaluate the summations to obtain the final expression for the mean square value of $Y_w(m)$:

$$\begin{aligned}
\mathbf{E}\{Y_w^2(m)\} &= T_s^2\sigma^4\left[M^2\rho^2(0) + 2M\sum_{k=-L}^{L}\rho^2(k)\cos^2(2\pi mk/M)\right] \\
&= [MT_s\sigma^2\rho(0)]^2 + 2MT_s^2\sigma^4\sum_{k=-L}^{L}\rho^2(k)\cos^2(2\pi mk/M)
\end{aligned}$$

Similarly, the mean square value of $Y_{cross}(m)$ is computed by squaring both sides of (2.10) and then taking expectation to yield

$$\mathbf{E}\{Y_{cross}^2(m)\}$$

$$= 4T_s^2 \sum_{n=-L}^{L} \sum_{n'=-L}^{L} \sum_{k=-L}^{L} \sum_{k'=-L}^{L} [\rho(k)\rho(k')s(n+k)s(n'+k')\cos(2\pi mk/M)\cos(2\pi mk'/M)$$

$$\bullet\, \mathbf{E}\{w(n-k)w(n'-k')\}]$$

$$= 4T_s^2\sigma^2 \sum_{n=-L}^{L} \sum_{k=-L}^{L} \sum_{k'=-L}^{L} \rho(k)\rho(k')s(n+k)s(n-k+2k')\cos(2\pi mk/M)\cos(2\pi mk'/M)$$

Equation (2.17) becomes

$$\mathbf{E}\{Y_{noise}^2(m)\}$$

$$= [MT_2\sigma^2\rho(0)]^2 + 2MT_s^2\sigma^4 \sum_{k=-L}^{L} \rho^2(k)\cos^2(2\pi mk/M)$$

$$+4T_s^2\sigma^2 \sum_{n=-L}^{L} \sum_{k=-L}^{L} \sum_{k'=-L}^{L} \rho(k)\rho(k')s(n+k)s(n-k+2k')\cos(2\pi mk/M)\cos(2\pi mk'/M)$$

Now, since $\mathbf{E}\{Y_{noise}(m)\} = \mathbf{E}\{Y_w(m)\}$, we substitute (2.13) and above expression into (2.15) to yield the final expression for the variance of $Y(m)$:

$$\mathbf{Var}\{Y(m)\} = 2MT_s^2\sigma^4 \sum_{k=-L}^{L} \rho^2(k)\cos^2(2\pi mk/M)$$

$$+4T_s^2\sigma^2 \sum_{n=-L}^{L} \sum_{k=-L}^{L} \sum_{k'=-L}^{L} [\rho(k)\rho(k')s(n+k)s(n-k+2k') \bullet \cos(2\pi mk/M)\cos(2\pi mk'/M)] \quad (2.18)$$

Note that $s(n)$ is defined only for $|n| \le L$.

3. TFR Performance Measure

In the following, we define the signal-to-noise power ratio for the decision statistic, $Y(m)$, as

$$SNR_Y(m) = \frac{[\mathbf{E}\{Y(m)\}]^2}{\mathbf{Var}\{Y(m)\}} \tag{2.19}$$

where $\mathbf{E}\{Y(m)\}$ and $\mathbf{Var}\{Y(m)\}$ are given in (2.14) and (2.18), respectively. The ratio, $SNR_Y(m)$, is proposed here as a performance measure that could be used to analyze the performance of a TFR in detecting a carrier signal in the presence of AWGN. In computing $SNR_Y(m)$ for a carrier signal using different kernel functions, we would like to: (i) determine whether $SNR_Y(m)$ is a suitable measure for a particular TFR; (ii) determine the effect of AWGN on $SNR_Y(m)$; (iii) employ $SNR_Y(m)$ as a relative performance measure in choosing the best kernel.

In the next chapter, we examine the Wigner-Ville Distribution of a carrier signal to observe the effect that truncating a signal in time has on the TFR. In addition, the results of the mean and variance analysis performed in this chapter are applied to the WVD.

III. WIGNER-VILLE DISTRIBUTION OF A CARRIER SIGNAL

In this chapter, the Wigner-Ville Distribution (WVD) of a finite duration continuous-time carrier signal is presented. Then, the TFR for a finite length carrier signal sequence is computed and shown to be in close agreement with that derived for continuous-time analysis.

A. CONTINUOUS-TIME WIGNER-VILLE DISTRIBUTION

1. Carrier Signal Unbounded in Time

For the continuous-time signal, $x(t)$, the WVD is given by

$$TF(t,f) = \int_{-\infty}^{\infty} x(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi f\tau}\,d\tau \tag{3.1}$$

Suppose $x(t) = A\cos(2\pi f_c t)$ and is assumed to exist for all time. The WVD is computed by forming the product

$$\begin{aligned} x(t+\tau/2)x^*(t-\tau/2) &= A^2\cos(2\pi f_c(t+\tau/2))\cos(2\pi f_c(t-\tau/2)) \\ &= \frac{A^2}{2}[\cos(4\pi f_c t) + \cos(2\pi f_c \tau)] \end{aligned} \tag{3.2}$$

and Fourier transforming with respect to τ :

$$TF(t,f) = \frac{A^2}{2}\cos(4\pi f_c t)\delta(f) + \frac{A^2}{4}[\delta(f-f_c) + \delta(f+f_c)],\ \forall t \tag{3.3}$$

where $\delta(\bullet)$ is the Dirac delta function. Note that the signal energy is concentrated at $f = f_c$, the center frequency of the carrier signal.

2. Finite Duration Carrier Signal

Consider the following carrier signal defined for t in the interval $[-T/2, T/2]$:

$$x(t) = A\cos(2\pi f_c t)\,rect(t/T)$$

where

$$rect(t/T) = \begin{cases} 1, & |t| \le T/2 \\ 0, & otherwise \end{cases}$$

We are interested in computing the closed form WVD for this signal in order to provide a means of comparison with the results computed using the discrete-time TFR. Towards this end, notice that

$$x(t+\tau/2)x^*(t-\tau/2) = \frac{A^2}{2}[\cos(4\pi f_c t) + \cos(2\pi f_c \tau)]rect\left(\frac{t+\tau/2}{T}\right)rect\left(\frac{t-\tau/2}{T}\right)$$

and

$$rect\left(\frac{t+\tau/2}{T}\right)rect\left(\frac{t-\tau/2}{T}\right) = rect\left(\frac{\tau}{2T\Lambda_T(t)}\right) \tag{3.4}$$

where

$$\Lambda_T(t) = \begin{cases} 1-\dfrac{2|t|}{T}, & |t| \le T/2 \\ 0, & otherwise \end{cases}$$

is a triangular-shaped function of duration T seconds centered at $t = 0$. Equation (3.4) defines a diamond-shaped support region in the (t,τ)-plane with unity value and has a Fourier transform pair given by

$$rect\left(\frac{\tau}{2T\Lambda_T(t)}\right) \leftrightarrow 2T\Lambda_T(t)sinc[2fT\Lambda_T(t)] \qquad \text{where } sinc(x) = \frac{\sin \pi x}{\pi x}$$

If we use the above transform pair and the modulation theorem, the Fourier transform of $x(t+\tau/2)x^*(t-\tau/2)$ with respect to τ becomes

$$TF(t,f) = A^2 T \Lambda_T(t)\cos(4\pi f_c t) sinc[2fT\Lambda_T(t)]$$

$$+\frac{A^2T}{2}\Lambda_T(t)\{sinc[2T\Lambda_T(t)(f-f_c)] + sinc[2T\Lambda_T(t)(f+f_c)]\} \qquad (3.5)$$

which is the WVD of a finite duration carrier signal. Observe that (3.5) becomes (3.3) in the limit as $T \to \infty$ as expected. To illustrate (3.5), the WVD for a finite length carrier signal sequence is shown in Figure 3.1. The WVD is characterized by a *sinc* function

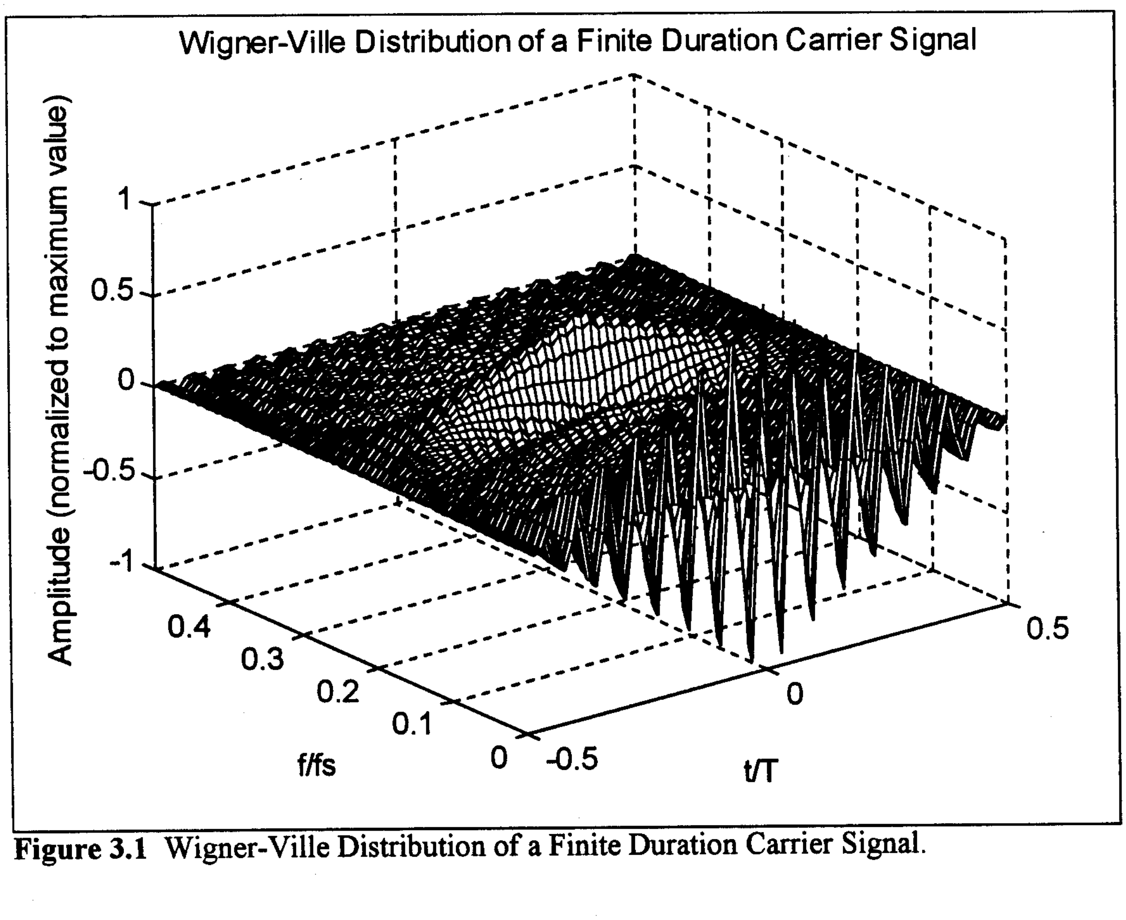

Figure 3.1 Wigner-Ville Distribution of a Finite Duration Carrier Signal.

centered at $f = f_c$ with a main lobe width of $2 \bullet [2T\Lambda_T(t)]^{-1}$ Hz with amplitude of $\frac{A^2T}{2}\Lambda_T(t)$. Note that the WVD depicted in Figure 3.1 has a triangular shape along the time axis (with maximum value at $t = 0$), and the *sinc* function centered at $f = f_c$ has a minimum main lobe width at $t = 0$ which increases towards infinite width as $|t| \to T/2$. Therefore, the WVD computed for a discrete-time sequence is consistent with that predicted by (3.5).

B. DISCRETE-TIME WIGNER-VILLE DISTRIBUTION

1. General

For a discrete-time sequence, the WVD kernel function is

$$\phi(n,k) = \delta(n), \quad \forall k$$

and if this is substituted into (2.2), we obtain the expression for the discrete WVD,

$$TF(n,m) = T_s \sum_{k=-L}^{L} x(n+k)x(n-k)\cos(2\pi mk / M) \tag{3.6}$$

2. Carrier Signal Sequence

If we set $x(n) = A\cos(2\pi m_0 n / M)$ for $|n| \le L$, (3.6) becomes

$$TF(n,m) = T_s \sum_{k=-L}^{L} A^2 \cos(2\pi m_0(n+k)/M)\cos(2\pi m_0(n-k))\cos(2\pi mk / M)$$

$$= \frac{A^2T_s}{2}\sum_{k=-L}^{L}[\cos(4\pi m_0 n / M) + \cos(4\pi m_0 k / M)]\cos(2\pi mk / M)$$

but since $-L \le n+k, n-k \le L$, we have

$$TF(n,m) = \frac{A^2 T_s}{2} \cos(4\pi m_0 n / M) \sum_{k=-L+|n|}^{L-|n|} \cos(2\pi mk / M)$$
$$+ \frac{A^2 T_s}{2} \sum_{k=-L+|n|}^{L-|n|} \cos(4\pi m_0 k / M) \cos(2\pi mk / M) \tag{3.7}$$

for $n,m = [-L, -L+1, \ldots, 0, \ldots, L]$, and $TF(n,m) = 0$ for n,m otherwise.

C. MEAN AND VARIANCE ANALYSIS FOR THE WVD

For the WVD, the kernel function $\phi(n,k) = \delta(n) \ \forall k$, and (2.5) yields

$$\rho(k) = \sum_{p=-L}^{L} \delta(p) = 1, \quad \forall k$$

It follows from (2.4) that

$$Y(m) = T_s \sum_{n=-L}^{L} \sum_{k=-L}^{L} x(n+k) x(n-k) \cos(2\pi mk / M) \tag{3.8}$$

1. Mean Analysis

For $x(n) = s(n) + w(n)$, where $s(n)$ is a deterministic signal and $w(n)$ is an AWGN sequence with zero mean and variance of σ^2, the mean of $Y(m)$ is given by (2.14) as follows:

$$\mathbf{E}\{Y(m)\} = Y_s(m) + MT_s\sigma^2 \tag{3.9}$$

where $Y_s(m)$ is evaluated using (3.8) with $x(n) = s(n)$.

2. Variance Analysis

Similarly, the variance of $Y(m)$ for the WVD can be expressed by (2.18) by setting $\rho(k) = 1$ to yield the following:

$$\mathbf{Var}\{Y(m)\} = 2MT_s^2\sigma^4 \sum_{k=-L}^{L} \cos^2(2\pi mk / M)$$

$$+4T_s^2\sigma^2 \sum_{n=-L}^{L} \sum_{k=-L}^{L} \sum_{k'=-L}^{L} s(n+k)s(n-k+2k')\cos(2\pi mk / M)\cos(2\pi mk'/M) \quad (3.10)$$

Notice that even for the simple case of the WVD, the expression for the variance of $Y(m)$ still requires an evaluation of a triple summation. However, for the WVD, the first term on the right side of (3.10) can be represented in closed form using the Euler relation and applying the formula for evaluating a geometric summation.

In the next chapter, we evaluate the mean and variance expressions derived in Chapter II for different kernels in addition to computing the WVD (and $Y(m)$) for various noise levels.

IV. NUMERICAL RESULTS

In this chapter, the equations developed in Chapter II for a deterministic signal embedded in AWGN are implemented for the case where the signal of interest is a carrier signal. A comparison of the results using various kernel functions is made for various noise levels.

A. COMPUTATION OF THE TFR

The TFR for a discrete-time sequence is computed using (2.1). For this thesis, the kernel function, $\phi(n,k)$, is an even function in both n and k (i.e. $\phi(n,k)=\phi(n,-k)$ and $\phi(n,k)=\phi(-n,k)$) and only *real* signals are used. By setting

$$y(n,k)=\sum_{p=-L}^{L}\phi(p,k)x(n-p+k)x(n-p-k)$$

and substituting into (2.1), we obtain

$$\begin{aligned} TF(n,m) &= T_s\sum_{k=-L}^{L}y(n,k)e^{-j\left(\frac{2\pi}{M}\right)mk} \\ &= 2T_s\Re e\left\{\sum_{k=0}^{L}y'(n,k)e^{-j\left(\frac{2\pi}{M}\right)mk}\right\} \end{aligned} \tag{4.1}$$

where

$$y'(n,k)=\begin{cases}0.5y(n,k), & k=0\\ y(n,k), & otherwise\end{cases} \tag{4.2}$$

Equation (4.1) is evaluated using FFT's with respect to index k (for fixed n). Refer to the Appendix for further details on this procedure.

B. CARRIER SIGNAL SEQUENCE

Suppose $x(t) = \cos(2\pi f_c t)$ for $|t| \leq T/2$, and we choose to sample $x(t)$ at $8f_c$ Hz (8 samples/cycle). This yields the ratio, $f / f_s = 0.125$, which is defined as the normalized frequency. Setting $t = nT_s$, we have

$$
\begin{aligned}
x(nT_s) &= \cos(2\pi f_c nT_s) \\
&= \cos(2\pi (f_c / f_s) n) \\
&= \cos(\pi n / 4)
\end{aligned}
$$

as our carrier signal sequence, $x(n)$. Figure 4.1 is a plot of this sequence for $|n| \leq 31$. Notice that $x(n)$ represents a carrier signal of arbitrary duration ($T = 62 T_s$ seconds)

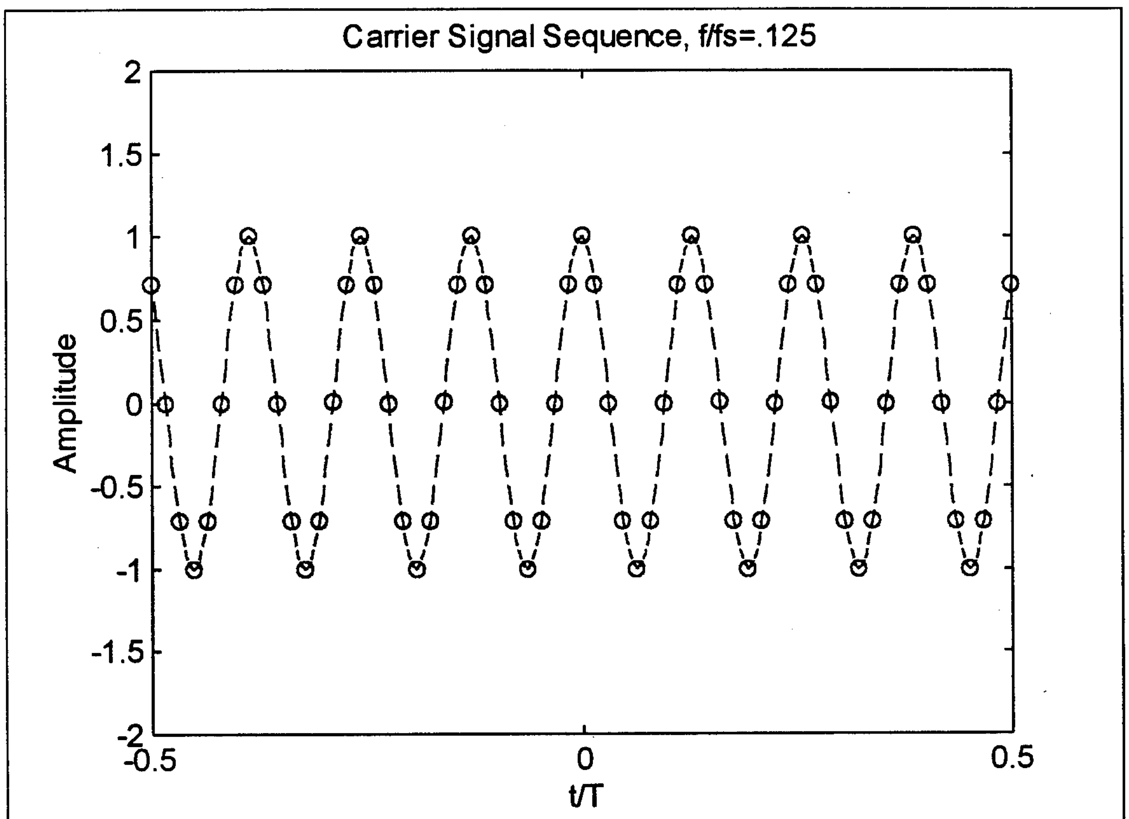

Figure 4.1 Example of a Carrier Signal Sequence.

which can be scaled to some real time sequence. For example, if a sampling rate of 8 kHz was used, $x(n)$ would represent a carrier signal with a center frequency of f_c=1 kHz with a duration of $T = 62/f_s = (62/8)$ ms (approximately 8 ms). For the remainder of this chapter, the sequence shown in Figure 4.1, specified by

$$x(n) = \cos(\pi n/4), \text{ for } |n| \le 31 \tag{4.3}$$

is that signal referred to as the "finite duration carrier signal" used for our analysis.

C. COMPUTATION OF THE TFR PERFORMANCE MEASURE

The TFR performance measure, $SNR_Y(m)$, is computed using (2.19). First, the mean and variance of $Y(m)$, given by (2.14) and (2.18), respectively, are computed. Without loss of generality, we set $T_s = 1$ in computing $SNR_Y(m)$. We have $M = 2(31)+1 = 63$, and we choose σ^2 values corresponding to "true" signal-to-noise power ratios of +5 dB, 0 dB, and -5 dB. Since the signal power is just one-half (0.5) for our signal, and σ^2 represents the noise power, the corresponding values are 0.1581, 0.5, and 1.581, respectively. Finally, we emphasize here that although $SNR_Y(m)$ is actually a sequence of bins indexed by m (with a total number of bins = 128) $SNR_Y(m)$ is plotted as a continuous function for visualization purposes.

1. Wigner-Ville Distribution

In Chapter III, the WVD for $x(n)$ defined in (4.3) is shown in Figure 3.1. If (2.3) is applied (i.e. summing the WVD over index n), we obtain our measure, $Y(m)$, which is shown in Figure 4.2. Notice that the maximum of $Y(m)$ occurs at twice f / f_s (i.e. maximum at $f / f_s = 0.25$). Recall from Chapter II that we expect frequencies to be multiplied by a factor of two as a result of employing the discrete-time TFR.

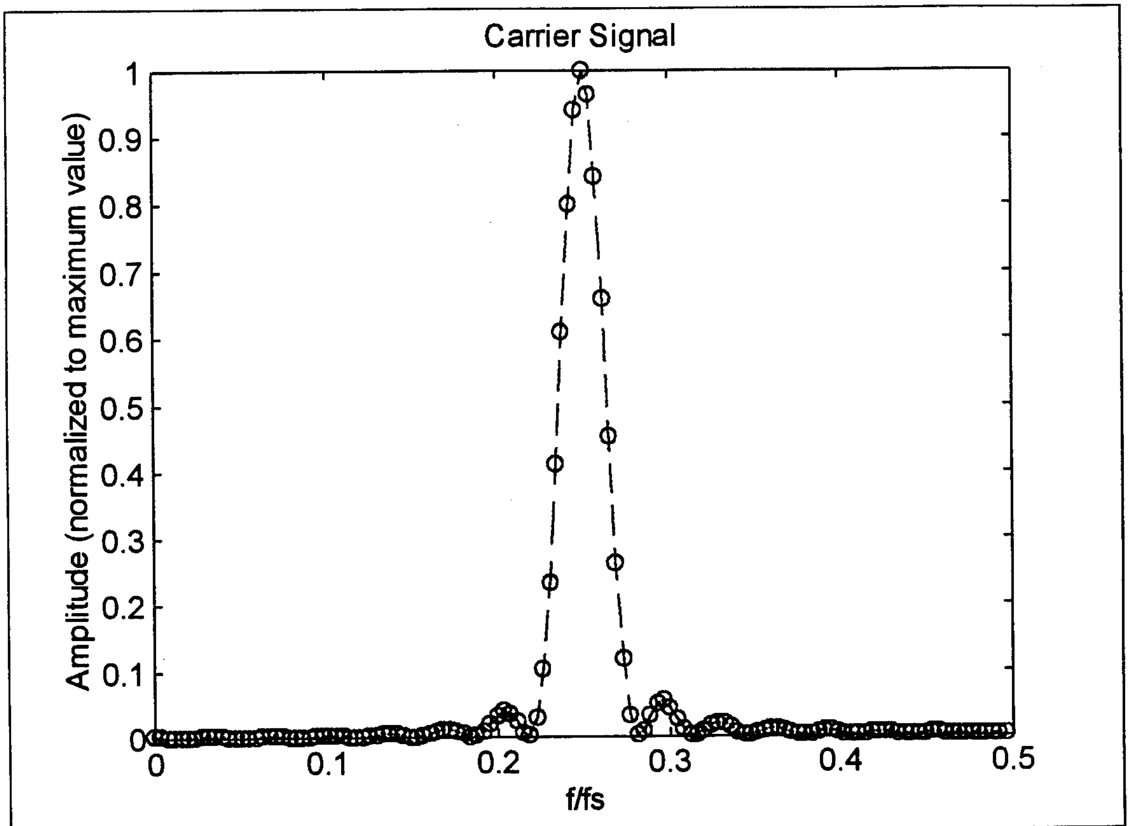

Figure 4.2 $Y(m)$ for the WVD of a Finite Duration Carrier Signal.

The $SNR_Y(m)$ for the WVD is shown in Figure 4.3. Note that $SNR_Y(m)$ for each noise level has been normalized to its maximum value in order to facilitate comparison between the peak level (at the bin corresponding to the center frequency) and all remaining levels. It is important to observe that $SNR_Y(m)$ is, in fact, maximized at the bin corresponding to the center frequency of the carrier signal. One would conclude that

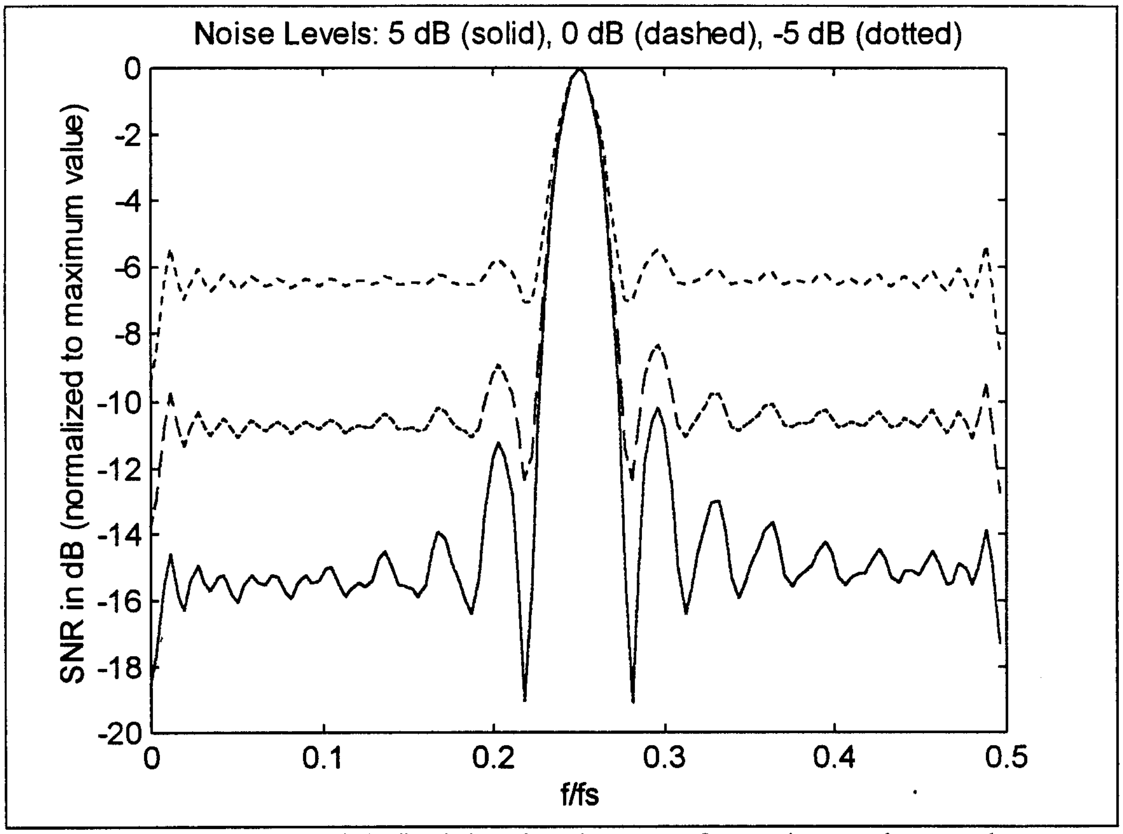

Figure 4.3 $SNR_Y(m)$ (in dB) Using the WVD for Various Noise Levels.

$SNR_Y(m)$ may be a suitable measure of the performance of the WVD in detecting a carrier signal in noise.

2. Comparison Among Several Kernels

In this section, $SNR_Y(m)$ is used as a means for comparing the performance of the WVD, the Choi-Williams Distribution (CWD), and the cone kernel TFR (CK-TFR) for detecting a carrier signal in the presence of AWGN. The three kernels are:

$$\text{WVD:}\ \ \phi(n,k) = \delta(n),$$

$$\text{CWD:}\ \ \phi(n,k) = e^{-\alpha n^2 k^2},$$

and

$$\text{CK-TFR:} \quad \phi(n,k) = \begin{cases} e^{-\alpha k^2}, & |n| \le |k| \\ 0, & otherwise \end{cases}$$

which is a cone-shaped support region in the (n,k)-plane. For each kernel, $|n| \le L$, and $|k| \le L$. We choose $\alpha = 0.000001$ for the CWD, and $\alpha = 0$ for the CK-TFR.

The decision statistic, $Y(m)$, computed for each kernel for the finite duration carrier signal defined in (4.3) is shown in Figure 4.4. Again, all plots are normalized to their maximum value for ease of comparison. Using Figure 4.4, we see that the CK-TFR yields a higher degree of frequency resolution as compared to that of the WVD and the CWD. In Figure 4.5, $SNR_Y(m)$ is plotted for the case where $\sigma^2 = 0.5$ (0 dB). Notice

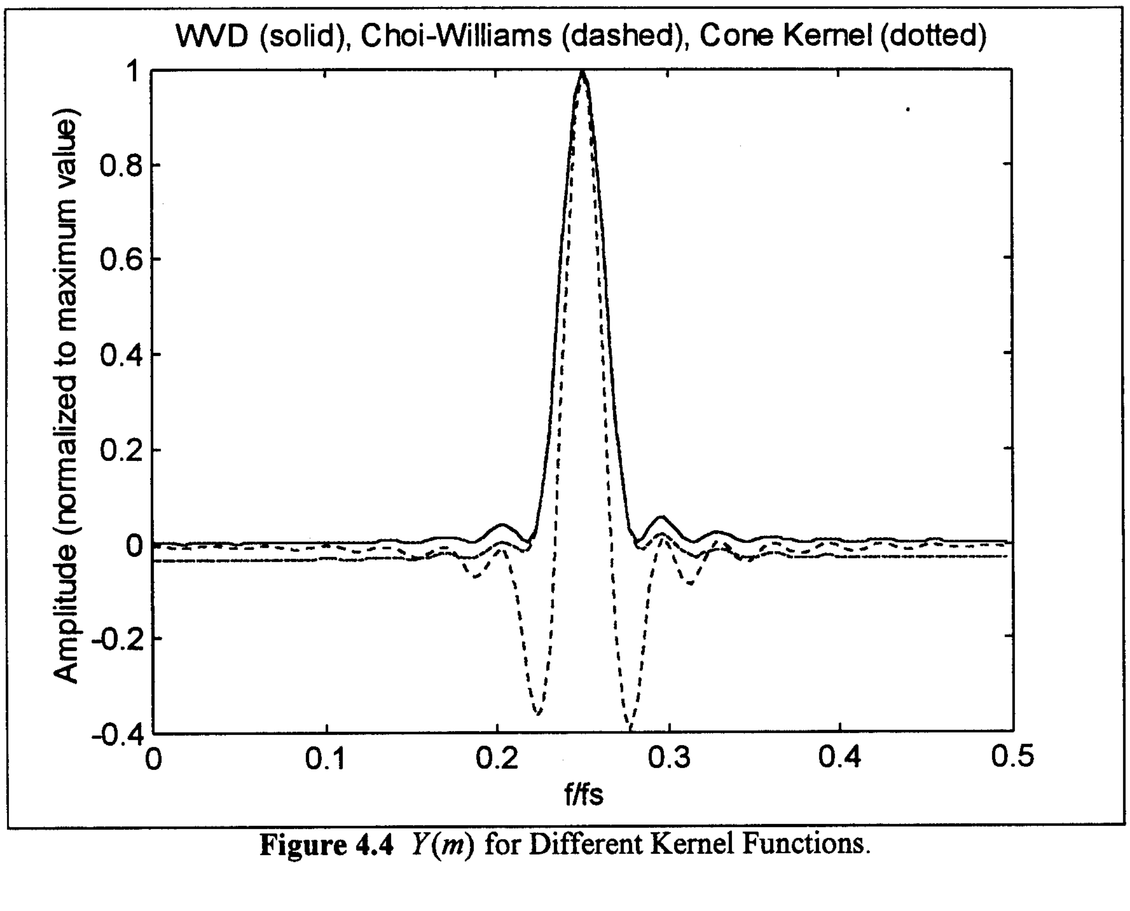

Figure 4.4 $Y(m)$ for Different Kernel Functions.

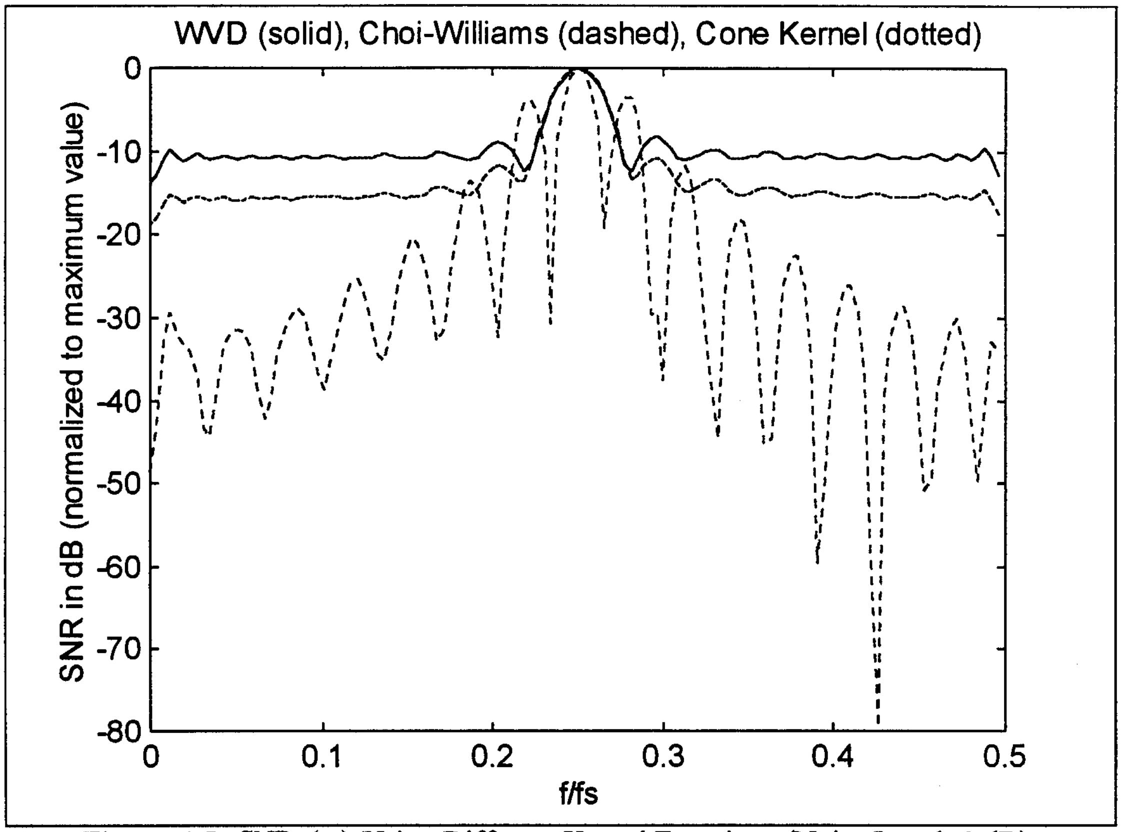

Figure 4.5 $SNR_Y(m)$ Using Different Kernel Functions (Noise Level: 0 dB).

that there are significant differences in the relative $SNR_Y(m)$ for main lobe (located at the center frequency) to sidelobe levels among the three kernels. Additionally, the *width* of the main lobe in $SNR_Y(m)$ is seen to be substantially affected by the selection of the kernel function. The cone kernel clearly has a much narrower main lobe width among the three kernels, where the CWD is seen to have the largest main lobe to sidelobe ratio. Note, however, that although the cone kernel has the smallest main lobe to sidelobe ratio, the majority of its sidelobes are *substantially* lower than the other two kernels. This seems to indicate that the cone kernel results in a higher concentration of signal energy in the vicinity of the center frequency. Since $SNR_Y(m)$ represents how the *average* signal-to-noise power ratio is distributed in frequency, one would expect the cone kernel to have the best chance in maximizing $Y(m)$ at or near the center frequency.

D. EXPERIMENTAL RESULTS FOR THE WVD

In the foregoing, the decision statistic, $Y(m)$, has been computed only for a deterministic signal. In this section, the signal defined in (4.3) is corrupted by an AWGN sequence of various noise levels corresponding to "true" signal-to-noise levels of +5 dB, 0 dB, and -5 dB. For each noise level, the WVD and $Y(m)$ are computed and shown in Figures 4.6 through 4.11. Note that although the WVD of the noisy signal becomes quite obscured as noise level increases, $Y(m)$ stills reveals a significantly higher measure of

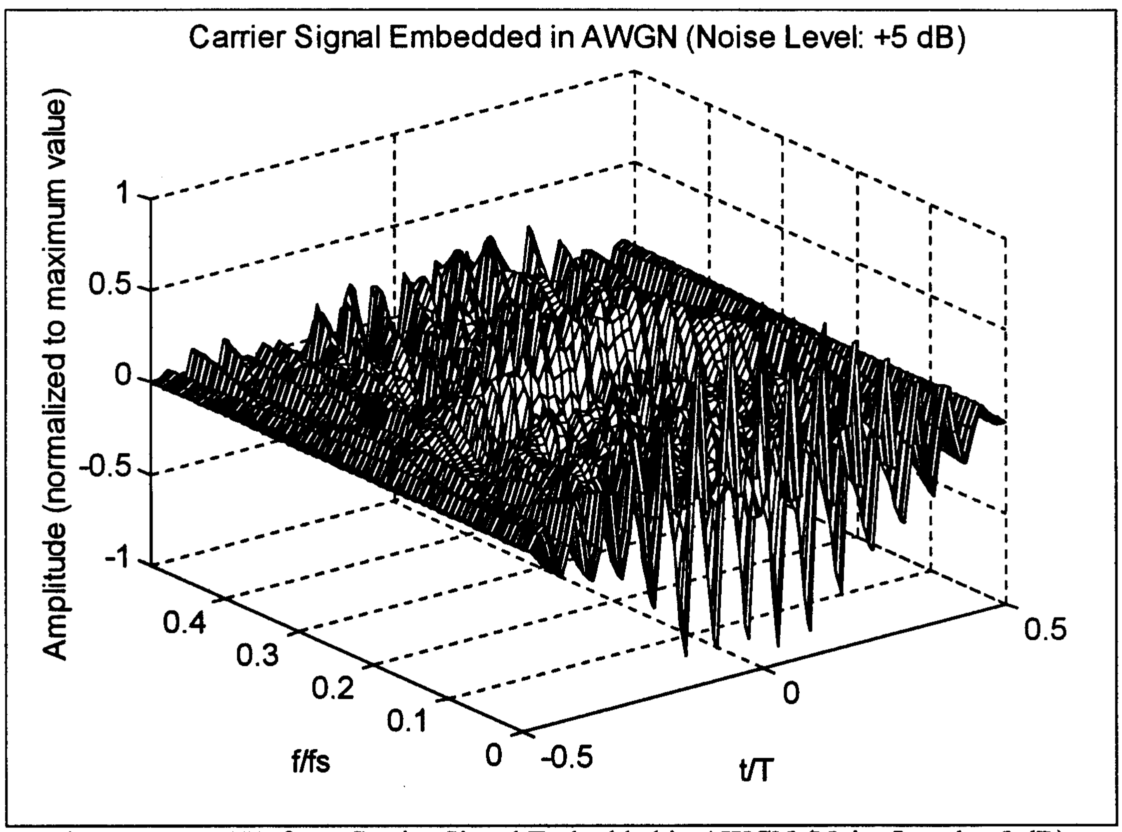

Figure 4.6 WVD for a Carrier Signal Embedded in AWGN (Noise Level: +5 dB).

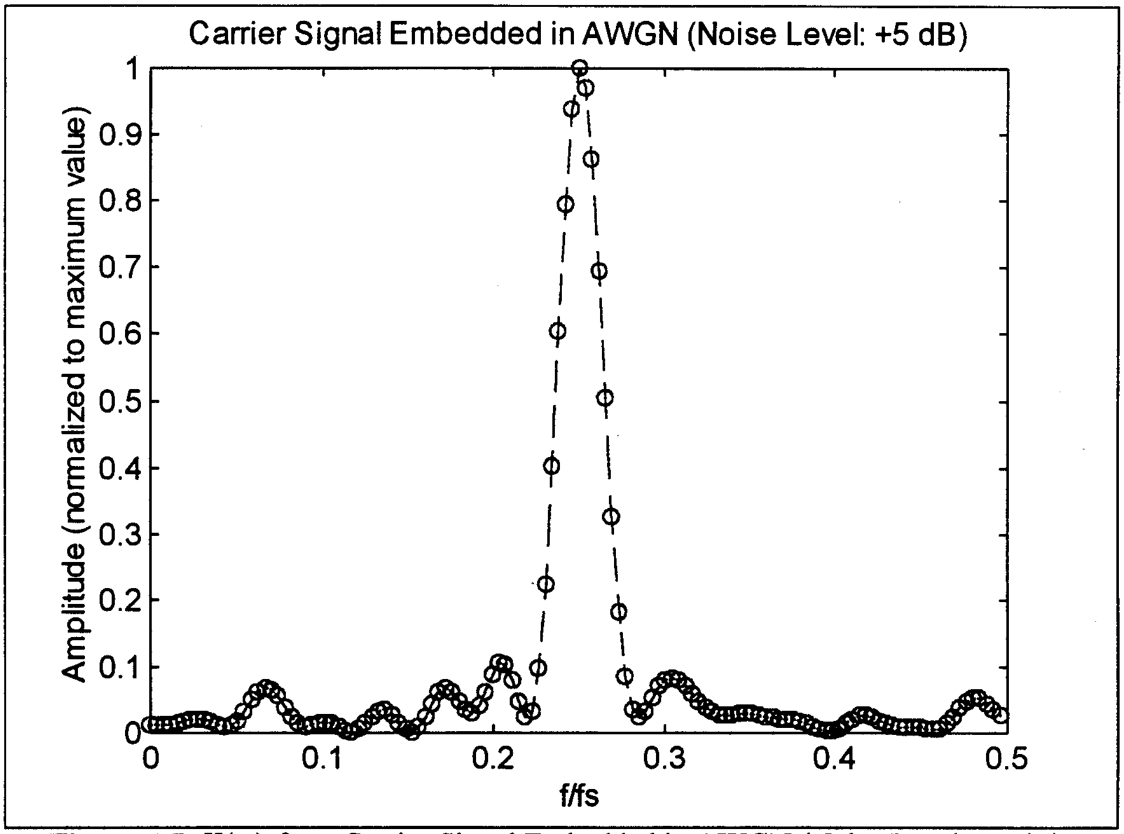

Figure 4.7 $Y(m)$ for a Carrier Signal Embedded in AWGN (Noise Level: +5 dB).

signal energy near the center frequency of the carrier signal. In fact, $Y(m)$ is maximized at the center frequency for these particular outcomes. These results, however, are only coincidental since $Y(m)$ is a random variable for each m.

In general, the maximum of $Y(m)$ may occur at locations other than at the center frequency, the likelihood of such occurrences increasing with noise level (σ^2). However, on the *average*, $Y(m)$ approaches the expected value of $Y(m)$ given by (3.9). In Figure 4.12, the average (sample mean) outcome of $Y(m)$ for 100 runs is plotted.

One might consider using the average value of $Y(m)$ as performance measure since it does tend to maximize $Y(m)$ at the center frequency. However, since $Y(m)$ is a random variable (for each m), one would expect the variance in $Y(m)$ to have a detrimental effect on the relative $SNR_Y(m)$ at the center frequency. In fact, this is the case and is seen in Figure 4.13 when contrasted with the mean of $Y(m)$ in Figure 4.12.

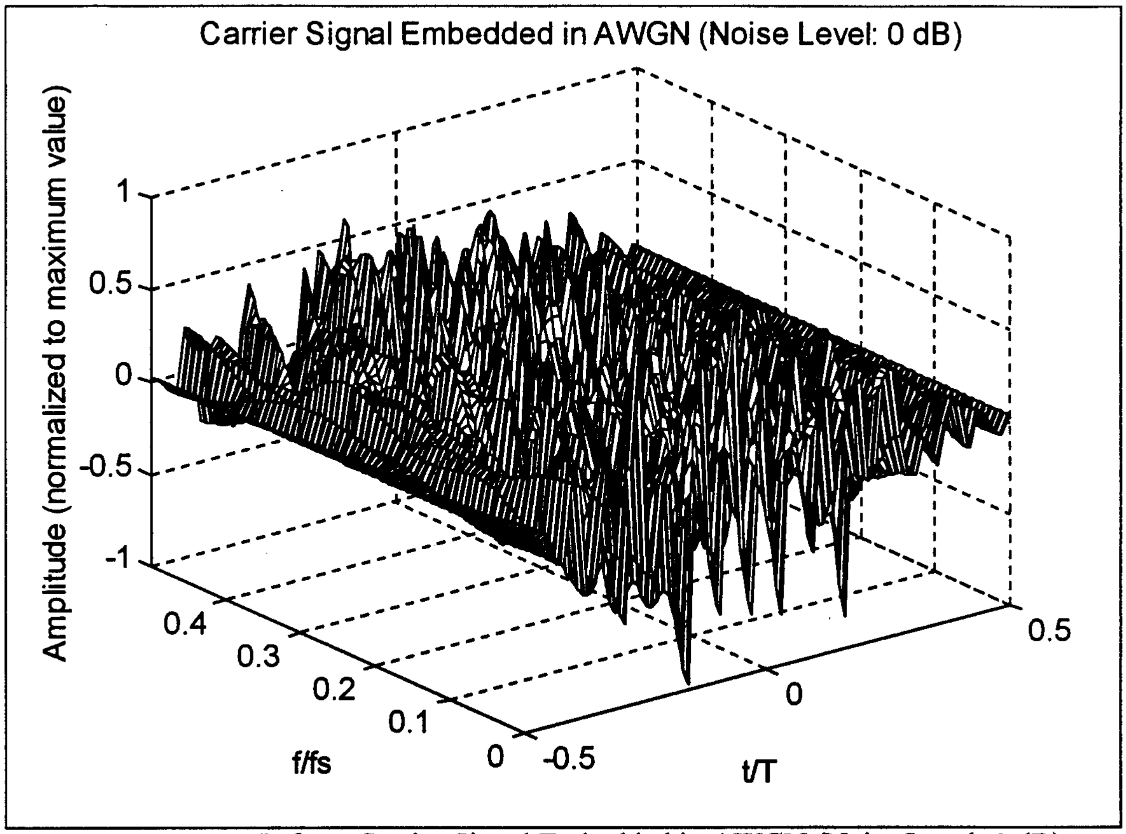

Figure 4.8 WVD for a Carrier Signal Embedded in AWGN (Noise Level: 0 dB).

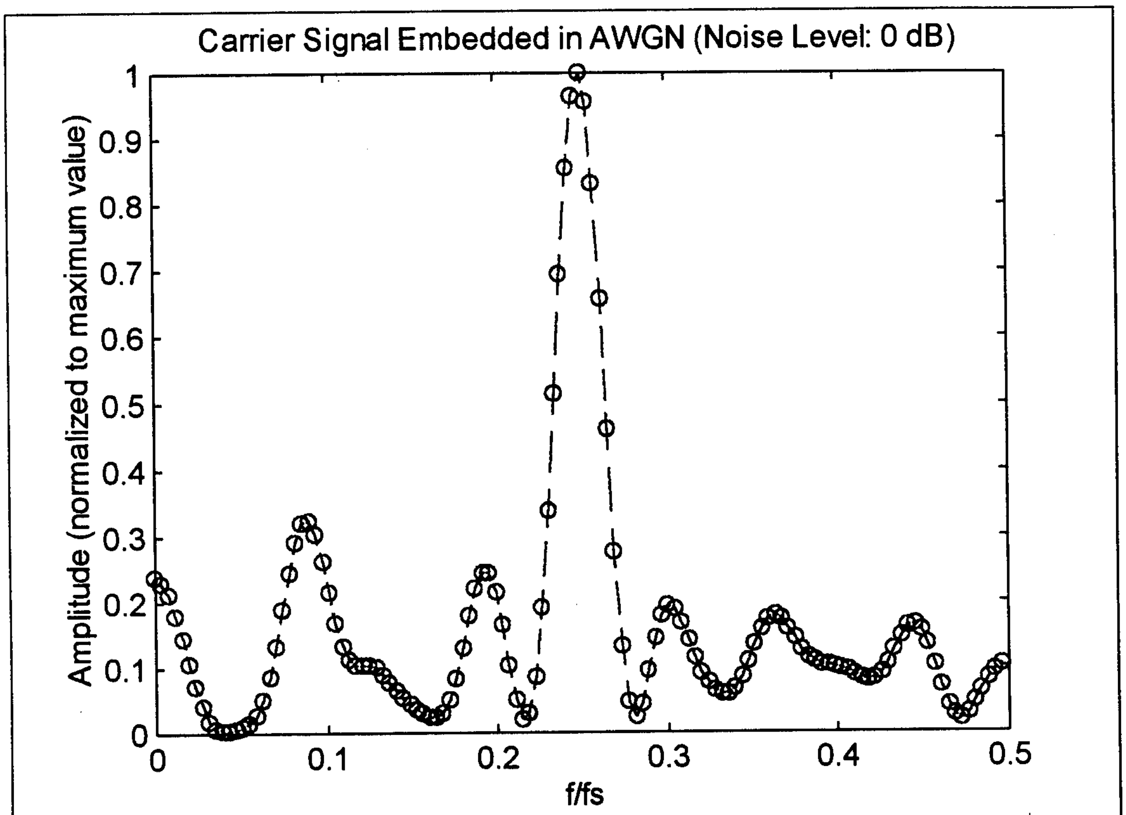

Figure 4.9 $Y(m)$ for a Carrier Signal Embedded in AWGN (Noise Level: 0 dB).

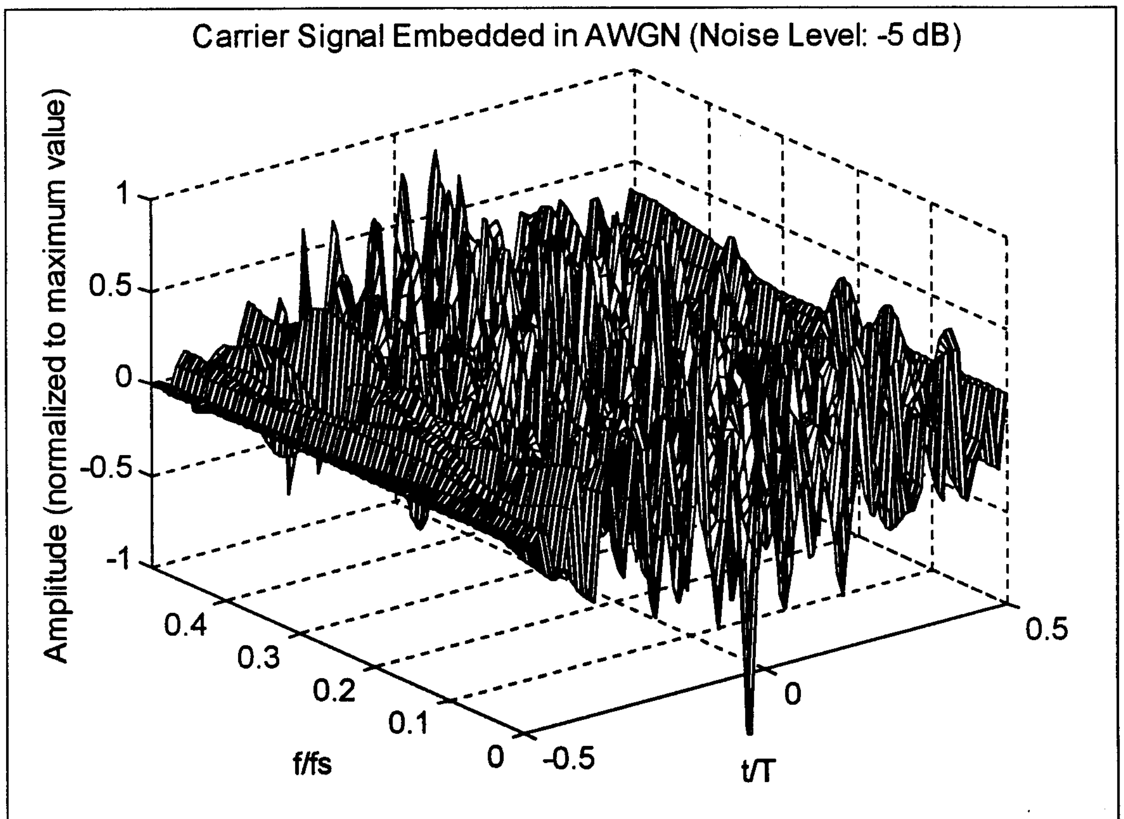

Figure 4.10 WVD for a Carrier Signal Embedded in AWGN (Noise Level: -5 dB).

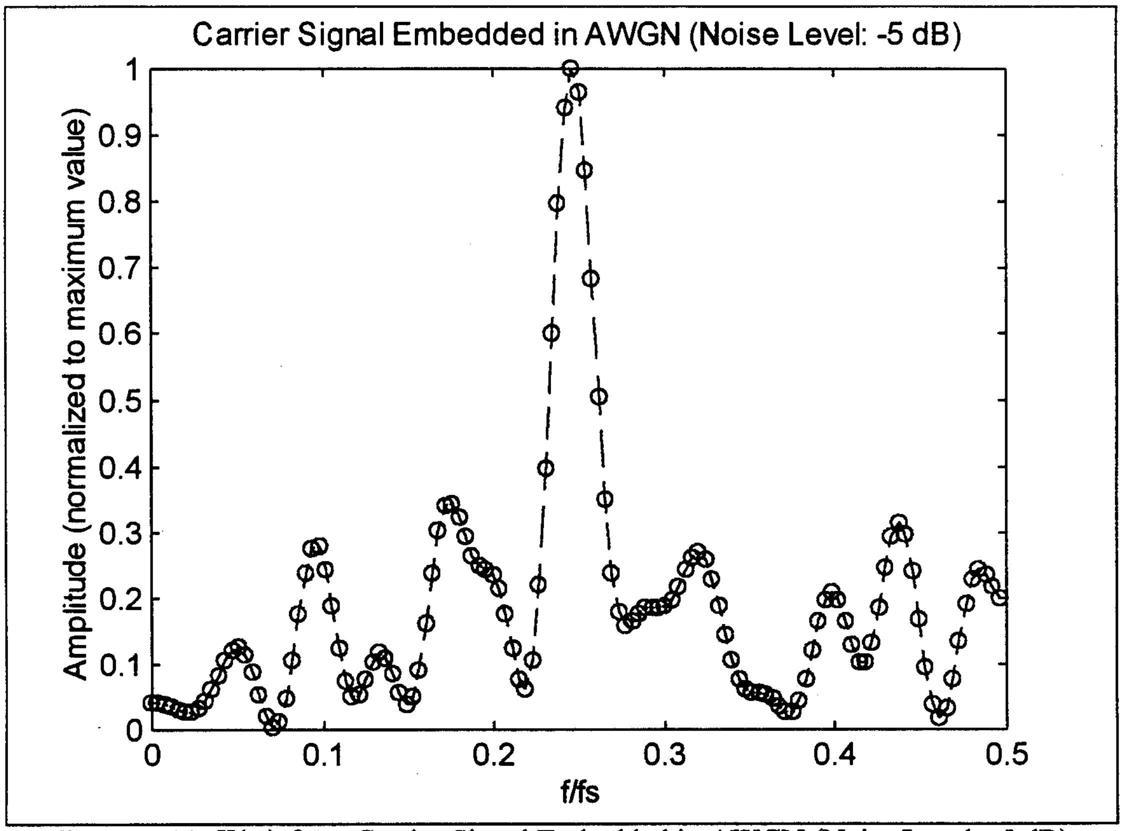

Figure 4.11 $Y(m)$ for a Carrier Signal Embedded in AWGN (Noise Level: -5 dB).

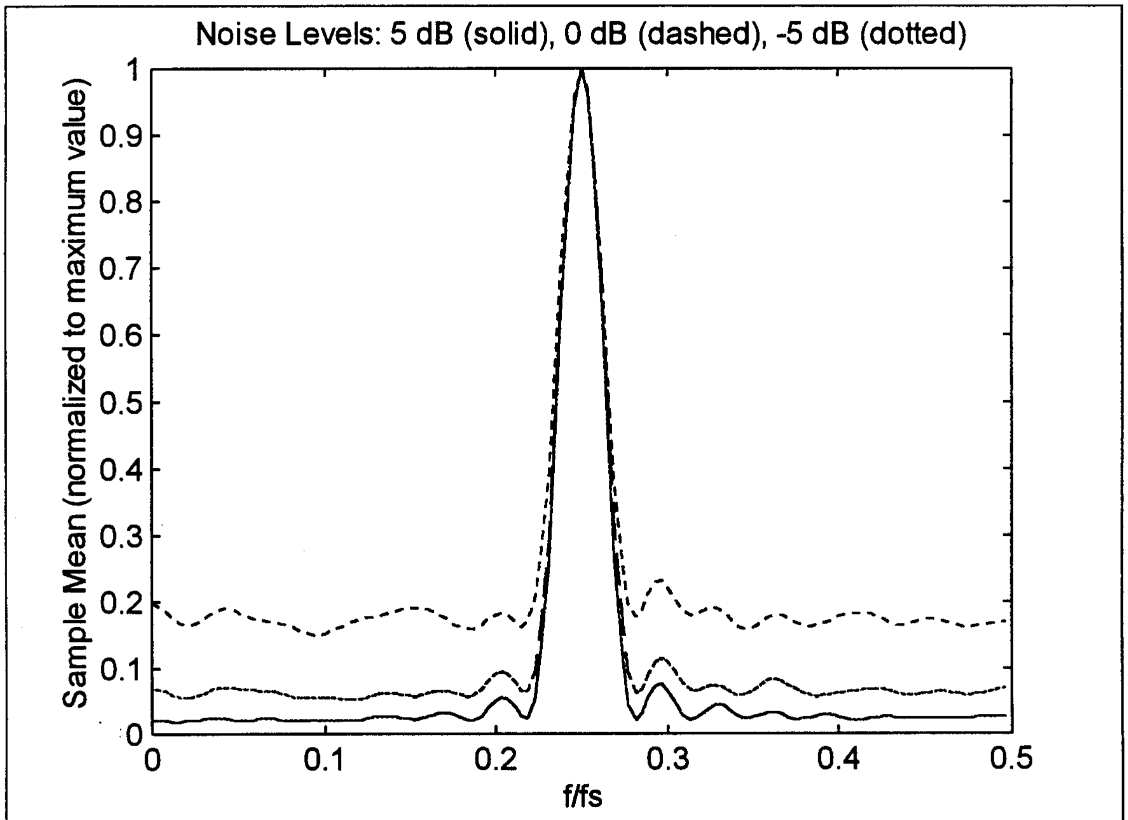

Figure 4.12 Estimated Mean of $Y(m)$ Using the WVD for Various Noise Levels.

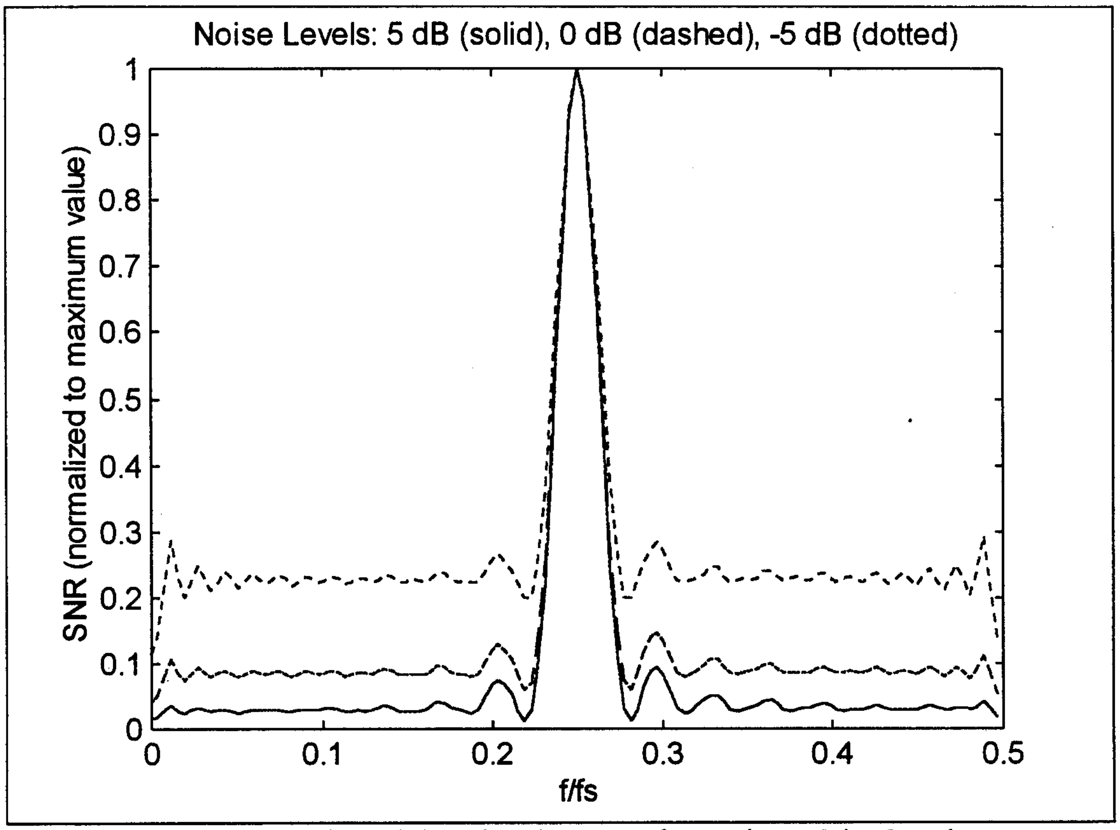

Figure 4.13 $SNR_Y(m)$ Using the WVD for Various Noise Levels.

V. CONCLUDING REMARKS

In this thesis, the signal detection problem was addressed using time frequency processing. Specifically, a measure (or decision statistic), $Y(m)$, computed from a TFR was proposed as a means of determining the center frequency of a carrier signal. The mean and variance of this measure were derived for the case of a signal embedded in AWGN. Furthermore, a signal-to-noise ratio, $SNR_Y(m)$, was proposed as a measure for the performance of a TFR in detecting a carrier signal in the presence of noise.

The Wigner-Ville Distribution (WVD) of a finite duration continuous-time carrier signal was derived in closed-form and shown to yield the WVD of an unlimited duration carrier in the limit as the signal duration approached infinite. The WVD was then computed for a finite length carrier signal sequence and shown to be consistent with the continuous-time analysis.

The decision statistic, $Y(m)$, and $SNR_Y(m)$ for the discrete-time WVD were then plotted for noise levels of +5 dB, 0 dB, and -5 dB (σ^2=.1581, 0.5 and 1.581, respectively). In all cases, both $Y(m)$ and $SNR_Y(m)$ were maximized at the center frequency. This serves as clear evidence that $Y(m)$ can be used to determine the center frequency of a carrier signal. Additionally, $SNR_Y(m)$ can also be used to predict the performance of the WVD based on the assumption of gaussian statistics in $Y(m)$.

A comparison of $Y(m)$ and $SNR_Y(m)$ computed for the WVD, the Choi-Williams Distribution (CWD), and the Cone Kernel TFR (CK-TFR) was made (assumed noise level of 0 dB for $SNR_Y(m)$ computation). A quantitative (based on the numerical results) analysis of $Y(m)$ revealed that the CK-TFR yields a higher degree of frequency resolution as compared to that of the WVD and CWD. The $SNR_Y(m)$ comparison seemed to suggest that using the cone kernel would maximize the chances of $Y(m)$ attaining a maximum value at or near the center frequency. This is based on the observation that $SNR_Y(m)$ represents how the *average* signal-to-noise power ratio is distributed in frequency. Since $SNR_Y(m)$ computed for the CK-TFR is highly concentrated in the vicinity of the center frequency, one would expect $Y(m)$ to have a maximum value located at or near the center frequency more often than it would if $SNR_Y(m)$ were distributed evenly in frequency.

Finally, the WVD and corresponding $Y(m)$ were computed for a noise corrupted signal at various noise levels (+5 dB, 0 dB, and -5 dB) to test whether the proposed measure yields a maximum value at the center frequency when the carrier is corrupted with noise. Even though the presence of the signal was barely discernible in the WVD for some noise levels, $Y(m)$ showed clear evidence of a signal present with energy concentrated at the center frequency.

In this thesis we have shown that the proposed measure, $Y(m)$, provides a means of detecting a carrier in the presence of noise. In the absence of noise, the center frequency of a carrier signal can easily be determined by inspection of the TFR. However, accomplishment of this task becomes nearly impossible when the carrier is corrupted with only a moderate level of noise. In addition, the TFR performance measure, $SNR_Y(m)$, can be used to compare the relative performance of different TFR's for detecting a carrier signal.

Additional research into the statistics of $Y(m)$ would enable one to perform a detection and false alarm analysis of a TFR when the frequency (bin) corresponding to the maximum value of $Y(m)$ is selected as the center frequency. Determination of the bit rate of a PSK signal might be made by examining the selected bin within the TFR over time for any periodicity. Ultimately, the proposed TFR measures, $Y(m)$ and $SNR_Y(m)$, might also provide a measure by which TFR's could be used to detect and discriminate among various digitally modulated signals such as PSK, FSK, and QAM signals.

APPENDIX. COMPUTATION OF THE TFR USING FFT'S

A. FFT FORMULATION OF THE TFR

We have from Chapter IV

$$TF(n,m) = T_s \sum_{k=-L}^{L} y(n,k) e^{-j\left(\frac{2\pi}{M}\right)mk}$$

$$= 2T_s \Re e\left\{ \sum_{k=0}^{L} y'(n,k) e^{-j\left(\frac{2\pi}{M}\right)mk} \right\}$$

where

$$y'(n,k) = \begin{cases} 0.5y(n,k), & k=0 \\ y(n,k), & otherwise \end{cases}$$

since $y(n,k) = \sum_{p=-L}^{L} \phi(n,k)x(n-p+k)x(n-p-k)$ is a *real and even* function in k. In order to evaluate (4.1) using FFT's, we represent (4.1) as a Discrete Fourier Transform (DFT) with respect to k (for each n) as follows:

$$TF(n,m) = 2T_s \Re e\left\{ \sum_{k=0}^{M-1} y''(n,k) e^{-j\left(\frac{2\pi}{M}\right)mk} \right\}$$

where

$$y''(n,k) = \begin{cases} y'(n,k), & 0 \le k \le L \\ 0, & otherwise \end{cases}$$

Observe that the above expression for $TF(n,m)$ is $2T_s$ times the real part of the DFT with respect to k (for each n) of $y''(n,k)$, which is a matrix of $M \text{x} M$ elements. Therefore, in order to use of the FFT to compute $TF(n,m)$, $y''(n,k)$ needs to have a number of rows (indexed by k) equal to an integer power of two. If we let $N = 2^i$ (i = positive integer) be the number of rows in $y''(n,k)$,

$$\Delta f = \frac{1}{NT_s} = \frac{f_s}{N}$$

is the frequency bin spacing and

$$f = m\Delta f = \frac{mf_s}{N}$$

is the frequency associated with each bin where $m = 0,1,\ldots,N/2-1$. Consequently, as a result of 'padding' the original $y'(n,k)$ in (4.1) with additional rows of zeros, the TFR is represented in frequency by a larger number of bins ($N/2$ bins). In effect, the true discrete-time TFR (which is continuous in frequency) remains the same but is sampled at smaller intervals (Δf) in frequency.

B. CHOOSING SAMPLING RATE

Since the TFR is to be computed using FFT's, the frequency bin spacing on the normalized discrete frequency axis (represented by f/f_s) is the inverse of a power of two (N^{-1}). In this thesis, we are interested in determining the center frequency of a carrier signal using the measure, $Y(m)$, defined in Chapter II.

In order for the maximum of $Y(m)$ to occur at the bin *exactly* corresponding to the carrier center frequency, the sample rate used to generate the sequence must be chosen such that Nf/f_s is an integer.

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